

Extending the Condorcet Criterion

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Over the last two centuries, voting theorists have developed a seemingly innocuous list of criteria for “good” voting systems. However, Arrow and Gibbard-Satterthwaite have shown that no voting system can satisfy all these criteria simultaneously; thus, in an election one must weigh the relative importance of each criterion. One criterion that some voting theorists have traditionally held holy is the Condorcet criterion. In the late eighteenth century, the Marquis de Condorcet proposed that, in an election consisting of more than two candidates, any candidate winning all head-to-head races should be the winner. Such a winner, if it exists, is called a Condorcet winner. While this criterion seems intuitive, it is a tricky criterion for voting systems to satisfy.

We question the logic of Condorcet’s criterion by generalizing and arguably strengthening the concept of a Condorcet winner. We define a Condorcet k -winner to be a candidate who wins every k -way (positional voting based) race with any $k - 1$ of the other candidates in the election. Condorcet k -winners, when they do exist, may differ for varying values of k . Our work has recently focused on finding and characterizing such Condorcet- k paradoxes.

Through the use of representation theory, we have found a useful decomposition of the profile space into irreducible submodules which correspond to different Condorcet- k races. We have uncovered a surprising influence of the Condorcet-2 space on all Condorcet- k outcomes for up to five candidate races. We have also determined the simultaneous existence and non-existence of certain collections of Condorcet- k winners for differing values of k for all elections with up to seven candidates.