CLAREMONT CENTER
for MATHEMATICAL SCIENCES

C C M S   C O L L O Q U I U M

Degenerate Diffusion in Heterogeneous Media

by

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Abstract: In this talk, I will present some recent results on the long-time behavior of non-negative solutions to the Cauchy problem for the Porous Medium Equation in the presence of variable density vanishing at infinity. More precisely, we consider the initial value problem

$$(\mathbf{P}) \qquad \begin{cases} \rho(x)\,\partial_t u = \Delta u^m & \text{in } \; Q := \mathbb{R}^n \times \mathbb{R}_+ \\ u(x,\,0) = u_0(x) & \text{in } \; \mathbb{R}^n \end{cases}$$

where we assume $m > 1$, $n \geq 3$ and $\rho(x)$ is positive, smooth and has a power-like decay at infinity, $\rho(x) \sim |x|^{-\gamma}$ as $|x| \to \infty$ for some $\gamma > 0$. The data u_0 are assumed to be non-negative and such that $\int_{\mathbb{R}^n} \rho(x) u_0(x)\,dx < \infty$, (finite-energy solutions).

For $m > 1$, the behavior of solutions depends on whether $0 < \gamma < 2$ or $\gamma > 2$. In the first case, the asymptotic behavior is described in terms of a one-parameter family of source-type, self-similar solutions of a related singular problem (so called Barenblatt-type solutions). For $\gamma > 2$, however, solutions to **(P)** have a *universal* long-time behavior in *separate variables*, typical of initial-boundary problems on bounded domains. If $\rho(x)$ has an intermediate decay, $\rho(x) \sim |x|^{-\gamma}$ with $2 < \gamma < \gamma_2 := N - (N-2)/m$, solutions still enjoy the finite propagation property (as in the case of lower γ). In this range a more precise description may be given at the diffusive scale in terms of the Barenblatt-type solutions $U_E(x,t)$ of the related singular equation $|x|^{-\gamma} u_t = \Delta u^m$.

Wednesday, October 2, 2013, at 4:15pm

Davidson Lecture Hall, Claremont McKenna College

Refreshments at 3:45 p.m. in CMC's Math Commons Room (Adams Hall 209) & wine and cheese after the talk in Math Commons Room (Adams Hall 209)

The dinner will be hosted by Prof. Alfonso Castro.
Please contact Prof. Castro if you are interested in attending the dinner